14

Waves

Main Course Contents

1. What is wave motion ? Give some important characteristics of wave motion.

[Chandigarh 03, 04 ; Himachal 09]

Ans. Wave motion. *Wave motion is a kind of disturbance which travels through a medium due to repeated vibrations of the particles of the medium about their mean positions, the disturbance being handed over from one particle to the next.* In a wave, both information and energy propagate from one point to another but there is no motion of matter as a whole through a medium.

Characteristics of wave motion :

1. In a wave motion, the disturbance travels forward through the medium while the particles simply vibrate about their mean positions.

2. Energy is transferred from one place to another without any net transport of material particles.

3. Each particle of the medium lags behind its predecessor by a definite phase angle.

4. Velocity of a particle is different from the velocity of propagation of the wave.

5. The wave velocity is constant in a given medium while particle velocity is different at different points.

2. *What are the different types of waves we come across ? Give examples of each type.*

Ans. Three basic types of waves :

(i) **Mechanical waves.** *The waves which require a material medium for their propagation are called mechanical waves or elastic waves.* For their propagation, the medium must possess the properties of inertia and elasticity.

Examples. Water waves, sound waves, seismic waves (waves produced during earthquake), etc.

(ii) **Electromagnetic waves.** *The waves which travel in the form of oscillating electric and magnetic fields are called electromagnetic waves. Such waves do not require any material medium for their propagation and are also called non-mechanical waves.* All electromagnetic waves travel through vacuum at the same speed, $c = 3 \times 10^8$ ms^{-1}.

Examples. Visible and ultraviolet light, radiowaves, microwaves, X-rays, etc.

(iii) **Matter waves.** *The waves associated with microscopic particles, such as electrons, protons, neutrons, atoms, molecules, etc., when they are in motion, are called matter waves or de-Broglie waves.*

Examples. Electron microscopes make use of the matter waves associated with fast moving electrons.

3. *Define transverse and longitudinal wave motions and explain them by diagrams.* [Himachal 04]

Ans. Two types of wave motion. Depending on the relationship between direction of oscillation of individual particles and direction of wave propagation, the mechanical waves can be transverse or longitudinal.

(i) **Transverse waves.** *These are the waves in which the individual particles of the medium oscillate perpendicular to the direction of wave propagation.* As shown in Fig. 14.1(a), consider a horizontal string with its one end fixed to a rigid support and other end held in the hand. If we give its free end a smart upward jerk, an upward kink or pulse is created there which travels along the string towards the fixed end. As shown in Fig. 14.1(b), if we continuously give up and down jerks to the free end of the string, a number of sinusoidal waves begin to travel along the string.
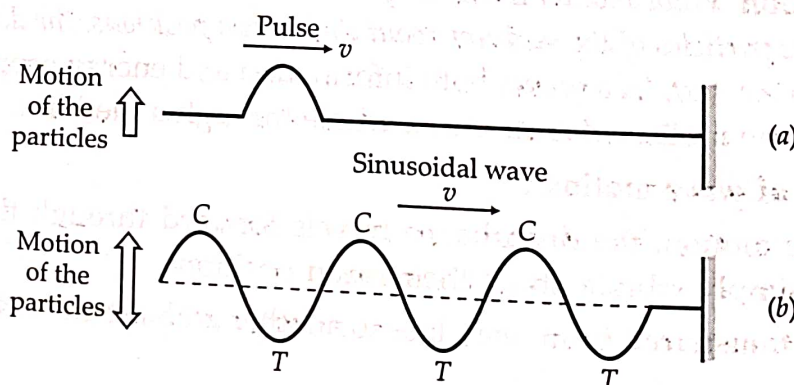

Figure 14.1 (a) A single pulse, (b) A sinusoidal wave sent along a stretched string.

Each part of the string vibrates up and down while the wave travels along the string. So the waves in the string are transverse in nature.

The points (C, C,.....) of maximum displacement in the upward direction are called crests. The points (T, T,.....) of maximum displacement in the downward direction are called troughs. One crest and one trough together form one wave.

(ii) **Longitudinal waves.** These are the waves in which the individual particles of the medium oscillate along the direction of wave propagation.

As shown in Fig. 14.2, consider a long hollow cylinder AB closed at one end and having a movable piston at the other end. If we suddenly move the piston rapidly towards right, a pulse of compression moves towards right and reaches the other end. Now if the piston is suddenly moved towards left, a pulse of refraction moves towards right.

If we continuously push and pull the piston alternately, a sinusoidal sound wave travels along the cylinder in the form of alternate compressions and rarefactions, marked C, R, C, R, etc. As the oscillations of an element of air are *parallel* to direction of wave propagation, the wave is a longitudinal wave. Hence sound waves produced in air are longitudinal waves.
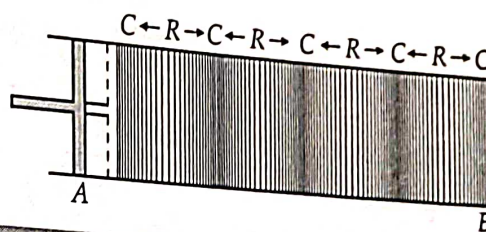

Figure 14.2 A sound wave produced in a cylinder by moving a piston back and forth.

4. *Mention the important properties which a medium must possess for the propagation of mechanical waves through it.*

Ans. **Essential properties of a medium for the propagation of mechanical waves.**

(i) *Elasticity.* The medium must possess elasticity so that the particles can return to their mean positions after being disturbed.

(ii) *Inertia.* The medium must possess inertia or mass so that its particles can store kinetic energy.

(iii) *Minimum friction.* The frictional force amongst the particles of the medium should be negligibly small so that they continue oscillating for a sufficiently long time.

5. *Through what type of media, can (i) transverse waves and (ii) longitudinal waves be transmitted ? Give reason.*

Ans. (i) **Media through which transverse waves can propagate.** Transverse waves travel in the form of crests and troughs. They involve changes in the shape of the medium. So they can be transmitted through media which sustain shearing stress, such as solids, strings and liquid surfaces.

(ii) **Media through which longitudinal waves can propagate.** Longitudinal waves travel in the form of compressions and rarefactions. They involve changes in volume and density of the medium. All media— solids, liquids and gases can sustain compressive stress, so longitudinal waves can be transmitted through all the three types of media.

Thus, both transverse and longitudinal waves can propagate through a steel bar while air can sustain only longitudinal waves.

6. Distinguish between transverse and longitudinal waves.

Ans.

	Transverse waves	Longitudinal waves
1.	The vibrations of the particles of the medium are perpendicular to the direction of propagation of the wave.	The vibrations of the particles of the medium are parallel to the direction of propagation of the wave.
2.	In transverse waves, alternate crests and troughs are formed.	In longitudinal waves, alternate zones of compression and rarefaction are formed.
3.	These waves may be formed in solids and over liquid surfaces.	These waves may be formed in solids, liquids and gases.
4.	These waves do not involve changes of pressure and density of the medium.	These waves involve changes of pressure and density of the medium.
5.	These waves can be polarised.	These waves cannot be polarised.
	Examples : (i) Movement of a kink on a rope. (ii) Waves set up on the surface of water.	Examples : (i) Sound waves in air. (ii) Vibrations of air column in an organ pipe.

7. In reference to a wave motion, define the terms (i) amplitude, (ii) time period, (iii) frequency, (iv) angular frequency, (v) wavelength, (vi) wave number, (vii) angular wave number and (viii) wave velocity.

Ans. Some definitions in connection with wave motion :

(i) **Amplitude.** *It is the maximum displacement suffered by the particles of the medium about their mean positions. It is denoted by* A.

(ii) **Time period.** *The time period of a wave is the time in which a particle of medium completes one vibration to and fro about its mean position. It is denoted by* T.

(iii) **Frequency.** *The frequency of a wave is the number of waves produced per unit time in the given medium.* It is equal to the number of oscillations completed per unit time by any particle of the medium. It is equal to the reciprocal of the time period T of the particle and is denoted by v. Thus

$$v = \frac{1}{T}.$$

SI unit of v is s^{-1} or hertz (Hz).

(iv) **Angular frequency.** *The rate of change of phase with time is called angular frequency of the wave.* It is clearly equal to $2\pi/T$, because the phase change in time T is 2π. It is denoted by ω. Thus

$$\omega = \frac{2\pi}{T} = 2\pi v$$

SI unit of $\omega = $ rad s^{-1}.

(v) **Wavelength.** *It is the distance covered by a wave during the time in which a particle of the medium completes one vibration to and fro about its mean position. Or, it is the distance between two nearest particles of the medium which are vibrating in the same phase.* It is denoted by λ.

Ans. Speed of a transverse wave on a stretched string. The speed of transverse wave through a stretched string is determined by *two* factors :

(i) Tension T in the string is a measure of elasticity in the string. Without tension no disturbance can propagate in the string.

$$\text{Dimensions of } T = [\text{Force}] = [\text{MLT}^{-2}]$$

(ii) Mass per unit length or *linear mass density* m of the string so that the string can store kinetic energy.

$$\text{Dimensions of } m = \frac{[\text{Mass}]}{[\text{Length}]} = [\text{ML}^{-1}]$$

Now, dimensions of ratio $\dfrac{T}{m} = \dfrac{[\text{MLT}^{-2}]}{[\text{ML}^{-1}]} = [L^2 T^{-2}]$

As the speed v has the dimensions $[LT^{-1}]$, so we can express v in terms of T and m as

$$v = C\sqrt{\frac{T}{m}}$$

From experiments, the dimensionless constant $C = 1$. Hence the speed of transverse waves on a stretched string is given by

$$v = \sqrt{\frac{T}{m}}$$

10. *On the basis of dimensional considerations, write the formula for the speed of transverse waves in a solid.*

Ans. Speed of transverse wave in a solid. The speed of transverse wave through a solid is determined by *two* factors : (i) Elasticity of shape or modulus of rigidity η of the solid. (ii) Mass per unit volume or density ρ determines its inertia. Now,

$$\text{Dimensions of ratio } \frac{\eta}{\rho} = \frac{[\text{ML}^{-1}\text{T}^{-2}]}{[\text{ML}^{-3}]} = [L^2 T^{-2}]$$

Dimensions of speed $v = [LT^{-1}]$

So we can express v in terms of η and ρ as : $v = C\sqrt{\dfrac{\eta}{\rho}}$

The dimensionless constant C is found to be unity. Thus the speed of transverse wave in a solid is given by

$$v = \sqrt{\frac{\eta}{\rho}}$$

11. *Write expressions for the speed of a longitudinal wave in (a) a liquid or gas, (b) an extended solid, and (c) a long solid rod.*

Ans. (a) Speed of a longitudinal wave in a liquid or gas. The speed of a longitudinal wave through a fluid is determined by *two* factors :

(i) The volume elasticity or bulk modulus κ of the fluid.

(ii) The density of the fluid which determines its inertia.

distance Δx in the positive X-direction. As the wave moves, each point of the moving waveform, such as point P marked on the peak retains its displacement y. This is possible only when the phase of the wave remains constant.

$\therefore$ $\omega t - kx = $ constant.

Differentiating both sides w.r.t., time t, we get

$$\omega - k\frac{dx}{dt} = 0 \quad \text{or} \quad \frac{dx}{dt} = \frac{\omega}{k}$$

But $\dfrac{dx}{dt} = $ wave velocity, v

$$\therefore \quad v = \frac{\omega}{k} = \frac{\lambda}{T} = v\lambda \quad \left[\because \omega = \frac{2\pi}{T}, k = \frac{2\pi}{\lambda}\right]$$
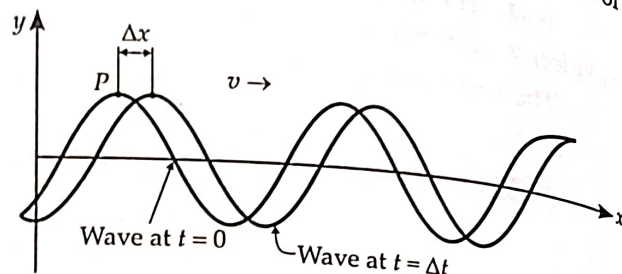

Figure 14.8 Plot of a harmonic wave at $t = 0$ and $t = \Delta t$.

18. *Explain the phenomenon of reflection of waves by considering a wave pulse travelling along a string, whose one end is (i) fixed to a rigid support and (ii) tied to a ring which can freely slide up and down a vertical rod. What are the phase changes in each case ?*

Ans. Reflection of a wave from a rigid boundary. As shown in Fig. 14.9, consider a wave pulse travelling along a string (rarer medium) attached to a rigid support, such as a wall (denser medium). Due to reaction of the wall, the incident crest is reflected back as a trough.

Hence *when a travelling wave is reflected from a rigid boundary, it is reflected back with a phase reversal or phase difference of π radians.*
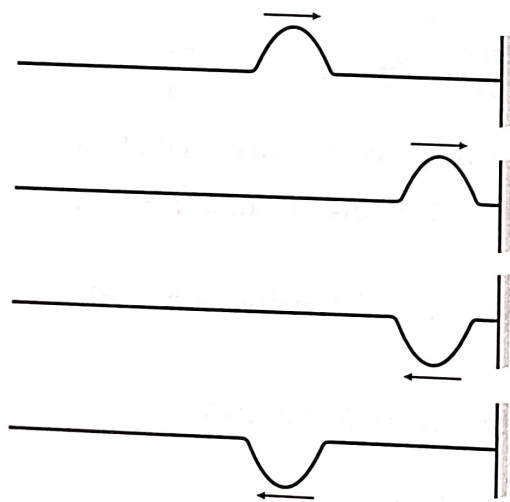

Figure 14.9 Reflection of a pulse in a string from a rigid support.
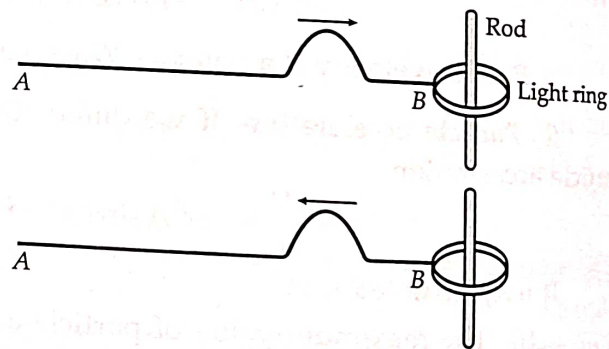

Figure 14.10   Reflection of a wave pulse on a string from a free boundary.

Reflection of a wave from an open boundary. As shown in Fig. 14.10, consider a wave pulse travelling along a string attached to a light ring, which slides without friction up and down a vertical rod. As the crest produced in the string at A reaches the end B, the ring moves up without offering any opposition. The crest is reflected as a crest.

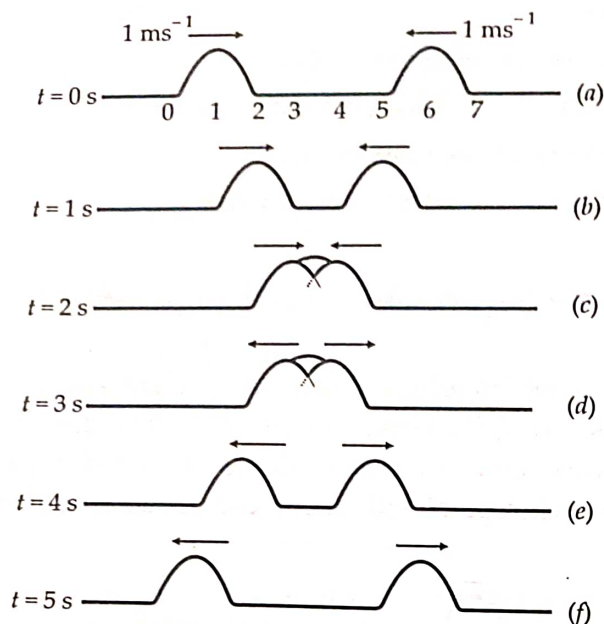Hence *when a travelling wave is reflected from a free or open boundary, it suffers no phase change.*

Figure 14.12 Superposition of two identical pulses travelling in opposite directions.
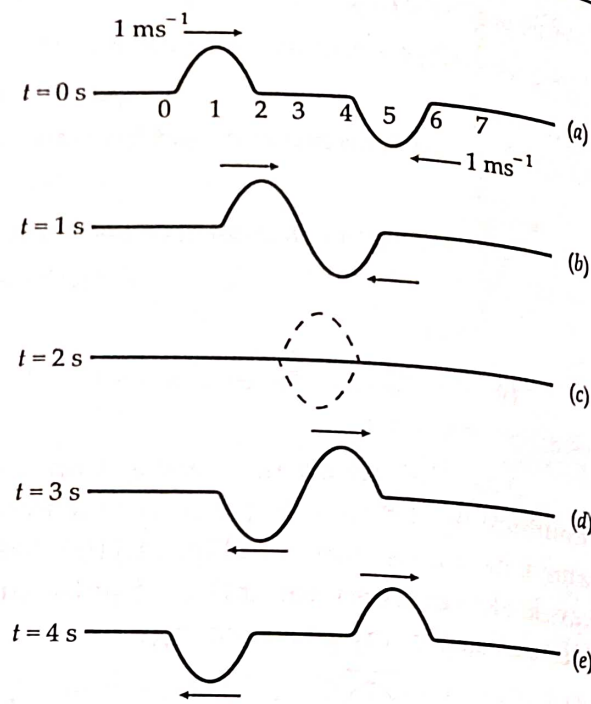

Figure 14.13 Superposition of two equal and opposite pulses travelling in opposite directions

(ii) Superposition of two pulses of equal and opposite shapes moving towards each other. Fig. 14.13 shows what happens when two equal and opposite pulses moving in opposite directions cross each other. When the positive pulse overlaps the negative pulse, they cancel each other. This is the case of *destructive interference.* The resultant displacement is the algebraic sum of the displacements of the individual waves.

The principle of superposition of waves leads to the phenomena of interference, beats and stationary waves.

21. (a) *What are stationary waves ?*

(b) *Why are stationary waves so called ?*

(c) *What is the necessary condition for the formation of stationary waves ?*

(d) *What are the two types of stationary waves ?*

Ans. (a) **Stationary waves.** *When two identical waves of same amplitude and frequency travelling in opposite directions with the same speed along the same path superpose each other, the resultant wave does not travel in the either direction and is called stationary or standing wave.* At some points, the particles of the medium always remain at rest. These are called *nodes.* At some other points, the amplitude of oscillation is maximum. These are called *antinodes.*

(b) In a standing wave, the resultant waveform does not move in either direction rather it keeps on repeating itself in the same fixed position. That is why, such waves are called *stationary* or *standing waves,* to distinguish them from *progressive* or *travelling waves* which travel through the medium with a definite speed v. In such waves, there is no transfer of energy along the medium in either direction.

For $n=1$, $\nu_1 = \dfrac{v}{4L} = \dfrac{1}{4L}\sqrt{\dfrac{\gamma P}{\rho}} = v$ (say)

This is the smallest frequency of the stationary waves produced in the closed pipe. It is called *fundamental frequency* or *first harmonic*.

For $n=2$, $\nu_2 = \dfrac{3v}{4L} = 3v$ (First overtone or third harmonic)

For $n=3$, $\nu_3 = \dfrac{5v}{4L} = 5v$ (Second overtone or fifth harmonic)

and so on. The various modes of vibration of a closed pipe are shown in Fig. 14.18.

34. (a) What are beats ? [Central Schools 08, 09]

(b) The beats are not heard if the difference in frequencies of the two sounding notes is more than 10. Why ? (Himachal 06)

Ans. Beats. When two sound waves of slightly different frequencies travelling along the same path in the same direction in a medium superpose upon each other, the intensity of the resultant sound at any point in the medium rises and falls alternately with time. *These periodic variations in the intensity of sound caused by the superposition of two sound waves of slightly different frequencies are called beats.* One rise and one fall of intensity constitute one beat. The number of beats produced per second is called beat frequency.
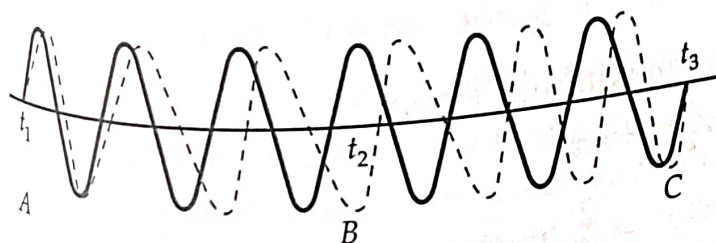
Beat frequency = Difference in frequencies of the two superposing waves
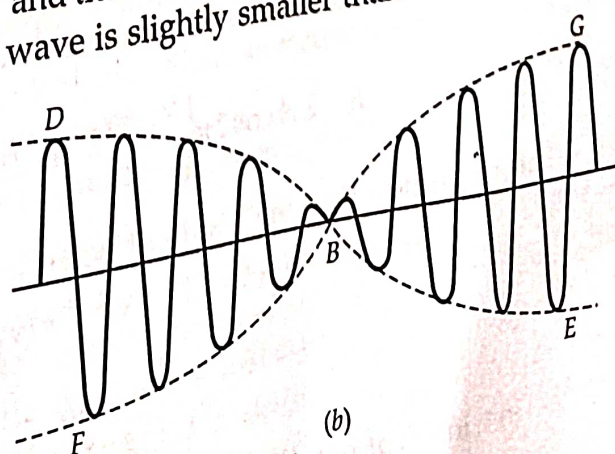
$$\nu_{beat} = \nu_1 - \nu_2$$

Essential condition for the formation of beats. For beats to be audible, the difference in the frequency of the two sound waves should not exceed 10. If the difference is more than 10, we shall hear more than 10 beats per second. But due to persistence of hearing, our ear is not able to distinguish between two sounds as separate if the time interval between them is less than (1/10)th of a second. Hence beats heard will not be distinct if the number of beats produced per second is more than 10.

35. Explain the formation of beats by graphical method.

Ans. Formation of beats by graphical method. In Fig. 14.19(a), the full line curve is the displacement-time curve of a wave of frequency ν_1 and the dashed curve is for a wave of frequency ν_2. Here ν_1 is slightly greater than ν_2, so the first wave is slightly smaller than second.

The number of beats produced per second is called beat frequency.

$$\nu_{beat} = \frac{1}{t_{beat}} \quad \text{or} \quad \nu_{beat} = \nu_1 - \nu_2$$

∴ Beat frequency = Difference between the frequencies of two superposing waves.

37. How will you experimentally demonstrate the phenomenon of beats in sound ?

Ans. **Experimental demonstration of beats in sound.** As shown in Fig. 14.20, mount two ning forks A and B of exactly the same frequency on two sound boxes, placed with their open ds facing each other.
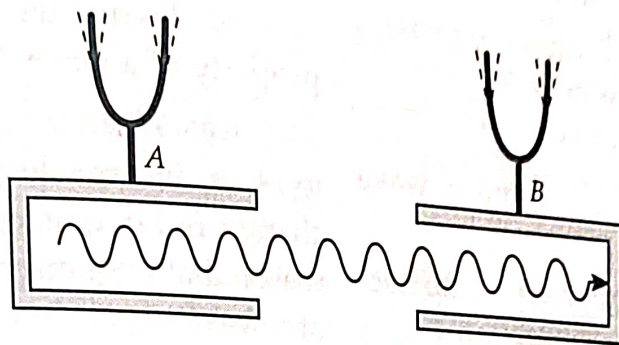

jure 14.20   Experimental demonstration of beats.

Now stick a little wax to the prong of one of them to as to slightly reduce its frequency. Set ne two tuning forks into vibrations. The two tuning forks will produce sound waves of different requencies. The intensity of the resulting sound will increase and decreasing periodically with time. Ve will actually hear beats.

38. Explain some practical applications of beats.

Ans. **Practical applications of beats :** (i) **Determination of an unknown frequency.** Suppose ν_1 is the known frequency of tuning fork A and ν_2 is the unknown frequency of tuning fork B When the two tuning forks are sounded together, suppose they produce b beats per second. Then

$$\nu_2 = \nu_1 + b \quad \text{or} \quad \nu_1 - b$$

The exact frequency may be determined by any of the following two methods :

(a) *Loading method.* Attach a little wax to the prong of the tuning fork B, its frequency decreases due to loading. Again the two tuning forks are sounded together.

   ☆ If the beat frequency *decreases* on loading, then $\nu_2 = \nu_1 + b$

   ☆ If the beat frequency *increases* on loading, then $\nu_2 = \nu_1 - b$

(b) *Filing method.* If a prong of the tuning fork B is filed, its frequency increases. Again, note the number of beats produced per second.

   ☆ If on filing the prong of B, the beat frequency decreases, then $\nu_2 = \nu_1 - b$

   ☆ If on filing the prong of B, the beat frequency increases, then $\nu_2 = \nu_1 + b$

(ii) **For tuning musical instruments.** Musicians use the beat phenomenon in tuning their musical instruments. If an instrument is sounded against a standard frequency and tuned until the beats disappear, then the instrument is in tune with the standard frequency.

(iii) **Use in electronics.** It is difficult to make low frequency oscillators. In practice, two high frequency oscillators with a small difference in their frequencies are used. Their low beat frequency ($\nu_1 - \nu_2$) serves the purpose of a low frequency oscillator.